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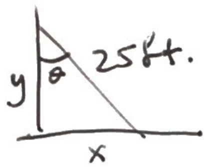
2.6 p 158 21, 23, 25, 27, 42, 43

2.7 p 161 1 – 23 odd, 27, 29, 31 – 33, 35 – 45 odd, 49, 57, 59, 61, 65, 67, 81, 83, 85

$\boxed{2.6}$ (21) ladder $\frac{dx}{dt} = 2$ ft/sec



(a) Find $\frac{dy}{dt}$ at $x = 7, 15, \&\ 24$ feet

$$x^2 + y^2 = 25^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = \frac{-2x}{y}$$

@ 7 feet: $\frac{-2(7)}{\sqrt{25^2-7^2}} = \frac{-14}{24} = \frac{-7}{12}$ ft/sec

@ 15 feet: $\frac{-2(15)}{\sqrt{25^2-15^2}} = \frac{-30}{20} = \frac{3}{2}$ ft/sec

@ 24 feet: $\frac{-2(24)}{\sqrt{25^2-24^2}} = \frac{-48}{7}$ feet/sec

(b) Find $\frac{dA}{dt}$ when $x = 7$, $y = 24$

$$A = \frac{1}{2}xy \leftarrow \text{product rule}$$

$$\frac{dA}{dt} = \frac{1}{2}x\left(\frac{dy}{dt}\right) + \frac{1}{2}\left(\frac{dx}{dt}\right)y$$

$$= \frac{1}{2}(7)\left(\frac{-7}{12}\right) + \frac{1}{2}(2)(24) = \frac{527}{24}\ \text{ft}^2/\text{sec}$$

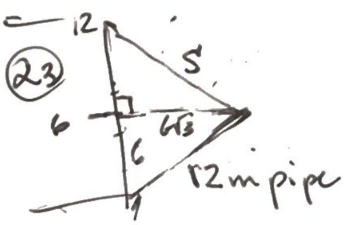
(c) find $\frac{d\theta}{dt}$ when $x = 7$ $y = 24$

$$\sin\theta = \frac{x}{25}$$

$$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{25}\left(\frac{dx}{dt}\right)$$

$$\left(\frac{d\theta}{dt}\right) = \frac{1\ (2)}{25\cos\theta} = \frac{2}{25\cdot\frac{24}{25}} = \frac{1}{12}\ \text{Rad/sec}$$

(23)



$\frac{ds}{dt} = -0.2$ m/sec find $\frac{dy}{dt}$ & $\frac{dx}{dt}$ when $y = 6$ m

$x = 6\sqrt{3}$

$s = 12$

$$x^2 + (12-y)^2 = s^2$$

$$\cancel{2}x\left(\frac{dx}{dt}\right) - \cancel{2}(12-y)\frac{dy}{dt} = \cancel{2}s\frac{ds}{dt}$$

Also $x^2 + y^2 = 12$ so

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0, \quad \frac{dx}{dt} = \frac{-y}{x}\frac{dy}{dt}$$

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23 con't.

Now substitute

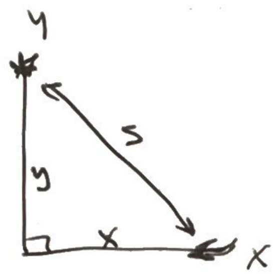
$$x\cdot\left(-\frac{y}{x}\frac{dy}{dt}\right) - (12-y)\frac{dy}{dt} = s\frac{ds}{dt}$$

$$\frac{dy}{dt}(-y-12+y) = 12(-.2)$$

$$\frac{dy}{dt} = \frac{12\ (-2)}{-12\ (10)} = \frac{1}{5} \text{ m/sec}$$

$$\frac{dx}{dt} = \frac{-6}{6\sqrt{3}}\left(\frac{1}{5}\right) = \frac{-1}{5\sqrt{3}} \text{ or } \frac{-\sqrt{3}}{15} \text{ m/sec}$$

(25)



$x = 225$ mi $\frac{dx}{dt} = -450$ m/hr

$y = 300$ mi $\frac{dy}{dt} = -600$ m/hr

(a) find $\frac{ds}{dt}$ $\quad x^2 + y^2 = s^2$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}s\frac{ds}{dt}$$

$$\frac{ds}{dt} = \frac{1}{s}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

$$= \frac{225\cdot{-450} + 300\cdot{-600}}{\sqrt{225^2 + 300^2}} \text{ mi/hr} = -750 \text{ m/hr}$$

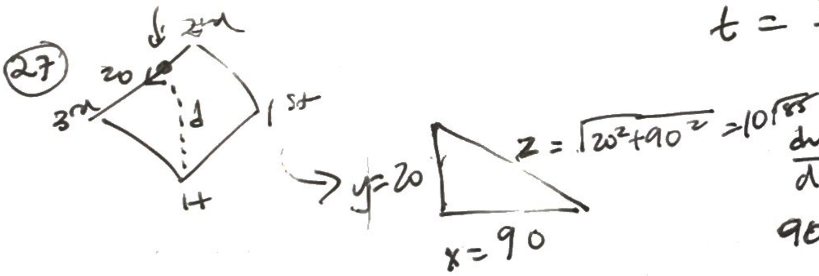
(b) What time will $s = 0$?

$rt = d$ so time =

$$|-750 \text{ m/h}|\ t = 375$$

$$t = \frac{375}{750} = \frac{1}{2} \text{ hr}$$

(27)



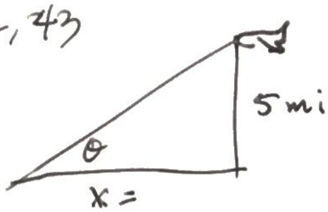
$\frac{dy}{dt} = -25$ ft/sec find $\frac{dz}{dt}$

$$90^2 + y^2 = z^2$$

$$0 + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt} = \frac{20}{10\sqrt{85}}(-25) = -5.42 \text{ ft/sec}$$

2.6
42, 43



find $\frac{d\theta}{dt}$ when $\frac{dx}{dt} = -600$ m/hr

for $\theta = \frac{\pi}{6}, \frac{\pi}{3}, 75°$

$$\tan\theta = \frac{5}{x} = 5x^{-1}$$

$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -\frac{5}{x^2}\left(\frac{dx}{dt}\right)$$

$$\frac{d\theta}{dt} = \cos^2\theta \ \frac{-5}{x^2}(-600 \text{ m/h})$$

$$= \cos^2\theta \ \frac{-5(-600)}{(5/\tan\theta)^2}$$

$$= \cos^2\theta \ \frac{600}{5}\tan^2\theta$$

$$= \cos^2\theta \ 120\tan^2\theta$$

when $\theta = \frac{\pi}{6}$ $\frac{d\theta}{dt} = \left(\frac{\sqrt3}{2}\right)^2 120\left(\frac{\sqrt3}{3}\right)^2 = 30$ rad/hr

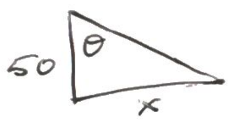
$\theta = \frac{\pi}{3}$ $\frac{d\theta}{dt} = \left(\frac{1}{2}\right)^2 120\left(\sqrt3\right)^2 = 90$ rad/h

$\theta = 75°$ $\frac{d\theta}{dt} = \left(\frac{\sqrt3 - 1}{2\sqrt2}\right)^2 120\left(2+\sqrt3\right)^2 = 60 + 30\sqrt3$ r/h

(43) 30 times a minute rotation

so $\frac{d\theta}{dt} = \frac{30 \cdot 2\pi}{1 \text{ min}} = 60\pi$ Rad/min

Find $\frac{dx}{dt}$ for $\theta = 30°, 60°, 70° = \frac{7\pi}{18}$



$$\tan\theta = \frac{x}{50}$$

$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} = 50\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{50 \cdot 60\pi}{\cos^2\theta} \text{ ft/min.}$$

$\theta = \frac{\pi}{6}$ $\frac{dx}{dt} = \frac{50 \cdot 60\pi}{60\left(\cos\frac{\pi}{6}\right)^2}$ ft/sec $= \frac{50\pi}{\left(\frac{\sqrt3}{2}\right)^2} = \frac{200\pi}{3}$ ft/sec

$\theta = \frac{\pi}{3}$ $\frac{dx}{dt} = \frac{50\pi}{\left(\frac{1}{2}\right)^2} = 200\pi$ ft/se

$\theta = \frac{7\pi}{18}$ $\frac{dx}{dt} = \frac{50\pi}{\cos^2\left(\frac{7\pi}{18}\right)} \approx 427.4316\pi$ ft/sec

2.R, p161 1-23 odd, 27, 29, 31-33, 35-45 odd, 49, 57, 59, 61, 65, 67, 81, 83, 85

① $f(x) = 12$ f' by limit process:

$$\lim_{h\to 0} \frac{f(h+x) - f(x)}{h} = \lim_{h \to 0} \frac{1}{h}(12 - 12) = 0$$

③ $f(x^3 - 2x + 1)$

$$\lim_{h\to 0} \frac{1}{h}\left((x+h)^3 - 2(x+h) + 1 - [x^2 - 2x + 1]\right)$$

$$\lim_{h\to 0} \frac{1}{h}\left(\cancel{x^3} + 3x^2h + 3xh^2 + h^2 - \cancel{2x} - 2h + \cancel{1} - \cancel{x^3} + \cancel{2x} - \cancel{1}\right)$$

$$\lim_{h\to 0} 3x^2 + 3xh + h - 2 = 3x^2 - 2$$

⑤ $g(x) = 2x^2 - 3x$ $c = 2$

$$\lim_{h\to 0} \frac{1}{h}\left(2(2+h)^2 - 3(2+h) - [2(2)^2 - 3(2)]\right)$$

$$\lim_{h\to 0} \frac{1}{h}\left(\cancel{8} + 8h + 2h^2 - \cancel{6} - 3h - \cancel{8} + \cancel{6}\right)$$

$$\lim_{h\to 0} \frac{1}{h}\left(5h + 2h^2\right) = \lim_{h\to 0} 5 + 2h = 5$$

⑦ $f(x) = (x-3)^{2/5}$

$$\lim_{x\to 3^-} f'(x) = \frac{2}{5}(x-3)^{-3/5} = -\infty$$

$$\lim_{x\to 3^+} f'(x) = +\infty$$

so not diff at $x = 3$
so ok: $(-\infty, 3) \cup (3, \infty)$

⑨ $y = 25$, $y' = 0$

⑪ $f(x) = x^3 - 11x^2$

$f'(x) = 3x^2 - 22x$

⑬ $h(x) = 6x^{1/2} + 3x^{1/3}$

$h'(x) = 3x^{-1/2} + x^{-2/3} = \frac{3}{\sqrt{x}} + \frac{1}{x^{2/3}}$

⑮ $y(t) = \frac{2}{3t^2} = \frac{2}{3}t^{-2}$

$g'(t) = -\frac{4}{3}t^{-3}$ or $\frac{-4}{3t^3}$

⑰ $f(\theta) = 4\theta - 5\sin\theta$

$f'(\theta) = 4 - 5\cos\theta$

⑲ $f(\theta) = 3\cos\theta - \sin\frac{\theta}{4}$

$f'(\theta) = -3\sin\theta - \frac{1}{4}\cos\theta$

2, R

(21) $f(x) = \frac{27}{x^3}$ find slope @ $(3,1)$

$f'(x) = \frac{27(-3)}{x^4}$ $\quad f'(3) = \frac{-3^4}{3^4} = -1$

(23) $f(x) = 4x^5 + 3x - \sin x$ slope @ $(0,0)$?

$f'(x) = 20x^4 + 3 - \cos x \Big|_{x=0} = 3 - 1 = 2$

(27) -30 ft/sec at $t=0$ $\quad s(t) = -16t^2 + v_0 t + s_0$ (see p116 for Ex)

600 feet

(a) position: $s(t) = -16t^2 - 30t + 600$

velocity: $s'(t) = -32t^2 - 30t$

(b) Avg velocity $t=1$ to $t=3$ $\quad \frac{\Delta s}{\Delta t} = \frac{s(3) - s(1)}{3-1} = -94$ ft/sec

(c) $s'(1) = -32(1)^2 - 30(1) = -62$ ft/sec

$s'(3) = -32(3)^2 - 30(3) = -126$ ft/sec

(d) Find t when $s(t) = 0$ on ground

$-16t^2 - 30t + 600 = 0$

$t = 5.258$ sec (using calc intersection)

(e) Find vel on impact: $s'(5.258) = -198.256$ ft/sec

(29) $f(x) = (5x^2+8)(x^2-4x-6)$

$f'(x) = (5x^2+8)(2x-4) + (10x)(x^2-4x-6)$

(31) $f(x) = (9x-1)\sin x$

$f'(x) = (9x-1)(\cos x) + 9\sin x$

(32) $f(t) = 2t^5 \cos t$

$f'(t) = 2t^5(-\sin t) + 10t(\cos t)$

(33) $f(t) = \frac{x^2+x-1}{x^2-1}$ $\quad f'(t) = \frac{(x^2-1)(2x+1) - (2x)(x^2+x-1)}{(x^2-1)^2}$

$= \frac{2x^3 + x^2 - 2x - 1 - 2x^3 - 2x^2 + 2x}{(x^2-1)^2}$

$= \frac{-x^2-1}{(x^2-1)^2}$

(35) $y = \frac{x^4}{\cos x}$

$y' = \frac{(\cos x)(4x^3) - x^4(-\sin x)}{\cos^2 x}$

2.R p161 -162

(37) $y = 3x^2 \sec x \qquad y' = 3x^2(\sec x \tan x) + 6x \sec x$

(39) $y = [x \cos x] - \sin x$

$y' = [x(-\sin x) + (1)\cos x] - \cos x = -x \sin x$

(41) $f(x) = (x+2)(x^3+5) = x^3 + 5x + 2x^2 + 10$

$f'(x) = 3x^2 + 5 + 4x \,\Big|_{x=-1} = 3 + 5 - 4 = 4$ is slope @ $(-1,6)$

tang. line $y - 6 = 4(x+1)$

(43) $f(x) = \dfrac{x+1}{x-1} \qquad f'(x) = \dfrac{(x-1)(1) - (1)(x+1)}{(x-1)^2} = \dfrac{-2}{(x-1)^2}\Big|_{x=\frac{1}{2}} = -8$

tangent line is $y + 3 = -8(x - \frac{1}{2})$

(45) $g(t) = -8t^3 - 5t + 12$

$g'(t) = -24t^2 - 5$

$g''(t) = -48t$

(49) $f(\theta) = 3 \tan\theta$

$f'(\theta) = 3\sec^2\theta$

$f''(\theta) = 6(\sec\theta \tan\theta)(\sec\theta)^1$

$= 6\sec^2\theta \tan\theta$

(57) $y = \dfrac{1}{(x^2+5)^3} \qquad y' = -3(x^2+5)^{-4}(2x) = \dfrac{-6x}{(x^2+5)^4}$

(59) $y = 5\cos(9x+1) \qquad y' = -5\sin(9x+1)(9)$

(61) $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4} \qquad y' = \dfrac{1}{2} - \dfrac{1}{4}(\cos 2x)(2) = \frac{1}{2}(1 - \cos 2x)$

(65) $f(x) = \left(\dfrac{x}{\sqrt{x+5}}\right)^3 \qquad f'(x) = 3\left(\dfrac{x}{\sqrt{x+5}}\right)^2 \left(\dfrac{\sqrt{x+5}\,(1) - x\left(\frac{1}{2}(x+5)^{-1/2}\right)}{x+5}\right)$

(67) $f(x) = \sqrt{1-x^3} \qquad f'(x) = \frac{1}{2}(1-x^3)^{-1/2}(-3x^2)\Big|_{x=-2} = -2$

the slope @ $(-2,3)$ is -2

(87) $x^3 y - x y^3 = 4$ (find $\frac{dy}{dx}$ implicitly)

$x^3\left(\frac{dy}{dx}\right) + y(3x^2) - \left[x\, 3y^2\left(\frac{dy}{dx}\right) + (1)y^3\right] = 0$

$\frac{dy}{dx}(x^3 - 3xy^2) = -3x^2y + y^3$

$\frac{dy}{dx} = \dfrac{y^3 - 3x^2y}{x^3 - 3xy^2}$

2.R p 162

88. $x \sin y = y \cos x$

$$x(\sin y)\left(\frac{dy}{dx}\right) + (1)\sin y = y(-\sin x) + (1)\cos x\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}(x \sin y - \cos x) = -\sin y - y \sin x$$

$$\frac{dy}{dx} = -\frac{\sin y + y \sin x}{x \sin y - \cos x} = \frac{\sin y + y \sin x}{\cos x - x \sin y}$$

85. Find Normal line to $(3,1)$

$$x^2 + y^2 = 10$$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-x}{y}\Bigg|_{(3,1)} = \frac{-3}{1} \quad \text{so } \perp \text{ slope } \frac{1}{3}$$

normal line $y - 1 = \frac{1}{3}(x-3)$